

## Thomson Scattering Signal and Uncertainty

The Alcator C-Mod incoherent Thomson scattering system has the following parameters.

|                                     |       |               |
|-------------------------------------|-------|---------------|
| Laser pulse energy                  | 1     | J             |
| Wavelength                          | 1.064 | $\mu\text{m}$ |
| Duration                            | 8     | ns            |
| Scattering Angle                    | 90    | deg           |
| Laser Beam Diameter                 | 3     | mm            |
| Scattering Length                   | 6     | mm            |
| Collection optics aperture diameter | 0.14  | m             |
| Optics distance from scattering vol | 1     | m             |
| Optical Transmission Efficiency     | 50    | %             |
| Detector Quantum Efficiency         | 20    | %             |

Using non-relativistic Thomson scattering approximations, estimate the following characteristics when scattering from a plasma of the following approximate (uniform) parameters:  $n_e = 2. \times 10^{20} \text{ m}^{-3}$ ,  $T_e = 2 \text{ keV}$ ,  $Z_{\text{eff}} = 1$ , diameter along viewing chord 0.4 m.

- The value of  $k\lambda_D$  (to check we are really in the incoherent regime).
- The total number of scattered photons detected over the entire scattered spectrum (for a single laser pulse).
- The fractional uncertainty in the measurement of plasma density resulting purely from scattered-photon statistics. [Hint: consult Appendix 2.]
- The spectral width of the scattered signal out to a frequency (or wavelength) displacement where the signal has fallen to  $e^{-2}$  of its peak intensity.
- The total number of plasma-light photons detected in this spectral width, during a time period of 8ns (the pulse duration), assuming that all the photons arise from bremsstrahlung.
- The number of plasma-light photons if the time period is 160 ns (to accommodate detector speed limitations) and the plasma emission is 10 times higher than bremsstrahlung (because of impurity radiation).
- The photon-statistical density-measurement fractional-uncertainty including plasma-light photons of case (f).

Since the non-relativistic spectral distribution has a Gaussian shape, one can use well-established statistical theorems to show that the standard-deviation in the measurement of its width from a sample of  $N$  photons is equal to the standard-deviation of the distribution times  $1/\sqrt{2(N-1)}$  (in the absence of noise photons). It is approximately a factor of  $\sqrt{1 + 4N_b/N}$  larger in the presence of a uniform background of  $N_b$  noise photons.

- What is the approximate fractional uncertainty in  $T_e$  measurement arising from photon statistics?