Quasineutral Cylindrical Probe Problem.

A long cylindrical floating langmuir probe of radius r_p resides in a plasma of which the ions are completely collisionless, have negligible energy far from the probe, and are thus attracted to the probe with purely radial velocity. The electron density is governed by the Boltzmann factor with temperature T_e. The Debye length is negligible with respect to the probe radius. The ions have charge Ze.

(a) Derive from basic principles the potential at which the quasi-neutral solution has infinite derivative. Take this value as the sheath edge.

(b) Obtain the full solution of the potential profile expressed implicitly in the form of a solution for radius r as a function of potential.

(c) From this solution, determine how the potential varies explicitly with r, asymptotically far from the probe.

(d) Derive an equation for the value of the probe potential in units of T_e/e, and solve it approximately when the ions are doubly ionized Helium ($Z = 2$, $m_i = 4m_{\mathrm{proton}}$).