Magnetics reconstruction problem.

A straight (screw or z-)pinch (tokamak surrogate) is diagnosed using magnetic flux ψ (per unit length along z in Webers) and magnetic field (in Tesla) measurements at four positions on the unit circle (lengths in meters). Taking the longitudinal coordinate to be z, along which everything is invariant, and x and y to be the coordinates in the plane of variation, the positions are

i	$x(i)$	$y(i)$
0	1.00	0.00
1	-1.00	0.00
2	0.00	1.00
3	0.00	-1.00

where i is the index of the measurement position.

You must devise, explain, and implement solution techniques that allow you to reconstruct the magnetic configuration and answer the questions posed for the following two situations.

(a) The interior of the unit circle can be considered to be a current-free region, prior to the appearance of the plasma. But there are external shaping fields applied by relatively distant coils. The measurements are

i	$\psi(i)$	$B_y(i)$	$-B_x(i)$
0	0.2851	0.4800	0.0480
1	0.1251	-0.3200	0.0480
2	-0.1469	0.0800	-0.3520
3	-0.2429	0.0800	0.4480

and you must reconstruct the fields inside the circle to determine:

(i) What is the magnetic field (B_x, B_y) at the center $(x, y) = (0, 0)$?

(ii) Is there a place in the circle where the total transverse field is zero $B_x = B_y = 0$, and if so where is it?

(b) The plasma is now present and can be considered to have current which is only in the z-direction, concentrated in a rather narrow filament, whose spatial extent is negligible as far as this measurement is concerned. The measurement values are:

i	$\psi(i)$	$B_y(i)$	$-B_x(i)$
0	0.7037	2.9077	0.0385
1	1.3038	-3.1072	0.0588
2	-0.9383	-0.3096	-0.8589
3	-1.0392	-0.2899	1.1562

(i) What is the value of the plasma current in Amps?

(ii) What is the position of the plasma current (centroid)?

(iii) Do the flux-surfaces inside the circle have any x-points? If so, where are they?

[Hint. In regions where there is no current, the flux satisfies $\nabla^2\psi = 0$. The problem calls for an approximate solution under appropriate assumptions. One way is to fit a small number of Fourier harmonics. You probably should write a little code to do the fitting. If you do, submit a print-out of your code with your solutions. If it is well commented, then it presumably serves the requirement of explanation as well as implementation.]