**Review of MHD Equilibrium**

**Force Balance:**
\[
\nabla p = j \times B
\n\]

Toroidal equilibrium requires us to satisfy this equation in both the minor- and major-radial directions.

1. **Minor-radial Force Balance.** (Cylindrical Tokamak)
   Approximate geometry as a straight cylinder. (Imagine torus cut at a certain \( \phi \) and then straightened out)

\[
\phi \approx \frac{\pi}{4}
\]
Take radial component of force balance equation:

\[ \frac{\partial p}{\partial r} = (\mathbf{j} \times \mathbf{B})_r = j_\theta B_z - j_z B_\theta \]

Also have \( \nabla \times \mathbf{B} = \mu_0 \mathbf{j} \). So

\[ \mu_0 j_z = \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) \quad , \quad \mu_0 j_\theta = -\frac{2}{r} B_z \]

Substitute

\[ \frac{\partial p}{\partial r} = \frac{1}{\mu_0} \left\{ -B_z \frac{\partial B_z}{\partial r} - \frac{B_\theta}{r} \frac{\partial (r B_\theta)}{\partial r} \right\} \]

So

\[ \frac{d}{dr} \left\{ p + \frac{B_\theta^2 + B_z^2}{2 \mu_0} \right\} + \frac{B_\theta^2}{\mu_0 r^2} = 0 \]

\[ \begin{array}{ccc}
\text{plasma pressure} & ; & \text{magnetic pressure} \\
\text{magnetic tension} & ; & \text{plasma kinetic pressure}
\end{array} \]

Definition of beta:

\[ \beta = \frac{2 \mu_0 p}{B_z^2} \]

This can be used in various ways, e.g. \( B_\theta \) or \( B_z \) or \( B \) & by using averages or peak.
Get a standardized form by integrating the force-balance equation \( \int_0^a r^2 \, dr \):

\[
\frac{df}{dr} r^2 \, dr = \left[ f r^2 \right]_0^a - \int_0^a f 2r \, dr \quad f = p + \frac{B_e^2}{2\mu_0}
\]

and also

\[
\int_0^a \frac{B_0}{r} \frac{d}{dr} (rB_0) \, r^2 \, dr = \int_0^a r B_0 \frac{d}{dr} (rB_0) \, dr
\]

\[
= \left[ \frac{(rB_0)^2}{2} \right]_0^a = \frac{a^2 B_0^2(a)}{2}
\]

So

\[
\int_0^a \left\{ \frac{d}{dr} \left( p + \frac{B_e^2}{2\mu_0} \right) + \frac{B_0}{\mu_0} \frac{d}{dr} (rB_0) \right\} r^2 \, dr
\]

\[
= a^2 p(a) + \frac{a^2 B_e^2(a)}{2\mu_0} - \int_0^a f 2r \, dr - \int_0^a \frac{B_e^2}{2\mu_0} 2r \, dr + \frac{a^2 B_0^2(a)}{2\mu_0} = 0
\]

Note that volume average of a quantity is

\[
\langle f \rangle \equiv \frac{\int_0^a f \, 2\pi r \, dr}{\int_0^a 2\pi r \, dr} = \frac{1}{a^2} \int_0^a f \, 2r \, dr.
\]

Hence equation can be written

\[
p(a) + \frac{B_e^2(a)}{2\mu_0} - \langle p \rangle - \langle \frac{B_e^2}{2\mu_0} \rangle + \frac{B_0^2(a)}{2\mu_0} = 0
\]

Usually assume \( p(a) \) is zero (or negligible) because \( n \& T \) are small at the edge. (Otherwise just redefine \( p \) by subtracting \( p(a) \).)
Then equation can be written:

\[
\frac{\langle p \rangle}{B_0^2(a) / 2\mu_0} = 1 + \frac{B_0^2(a) - \langle B_z^2 \rangle}{B_0^2(a)}
\]

Left hand side is a type of beta using average pressure and edge poloidal field. We call it beta poloidal:

\[
\beta_p \equiv \frac{2\mu_0 \langle p \rangle}{B_0^2(a)}
\]

Since \( B_0(a) = \frac{\mu_0 I_p}{2\pi a} \), it can also be written

\[
\beta_p = \frac{8\pi^2 a^2 \langle p \rangle}{\mu_0 I_p^2}
\]

so it is sometimes called \( \beta_i \).

If \( B_z \) were uniform then \( B_z^2(a) - \langle B_z^2 \rangle = 0 \) and we would require a definite pressure, \( \beta_p = 1 \). In general \( \beta_p \) is not equal to 1 (necessarily). Then \( B_z \) must vary across the plasma so as to satisfy force balance.

Average \( B_z^2 \) will be greater/less than \( B_z^2(a) \) according as \( (\beta_p - 1) \) is less/greater than zero. \( B_z \) smaller inside plasma \( \rightarrow \) diamagnetic \( B_z \) larger inside plasma \( \rightarrow \) paramagnetic.
\[ \beta_p < 1 \]
\text{Paramagnetic}

\[ \beta_p > 1 \]
\text{Diamagnetic}

Tokamak is characterized by having
\[ B_z(\phi) \Rightarrow B_\theta \]

as opposed to general screw pinch (stabilized Z-pinch, reversed field pinch etc) in which
\[ B_z \text{ may be } \sim B_\theta. \]

The changes in \( B_z \) for tokamak equilibrium are relatively small.

Remember the \( z \) is the toroidal direction \( (\phi) \)
\[ B_z = B_\phi = B_t \] in this approx.

Can't simplify to straight cylinder so equations are more difficult.
Can either calculate direct from force equations with toroidal geometry (Hutchinson, Principles of Plasma Diagnostics p.27-29) or from the Grad Shafranov equation (Freudberg, Ideal MHD p.107 ff).

**Heuristic approach.**
There are two basic self-forces tending to expand the plasma
1. Self magnetic force of plasma current
2. Pressure (tyre-tube) force.

(1) Self magnetic force on a wire loop can be calculated by the method of virtual work.
Suppose wire has self-inductance $L$ carrying current $I$.
If it expands an amount $\Delta R$
then any change in stored magnetic energy of the loop is equal to the work done by the outward force $F$ in distance $\Delta R$.

\[
F_{\text{m}} \Delta R \cdot 2\pi R = -\Delta W_m
\]

Force/length length $\uparrow$ change in mag. energy.
Assuming no circuits attached to wire & no resistive losses, the wire’s current is such that flux is conserved: \( \Delta I \cdot L = \text{const.} \)

\[ \Delta \Phi = \Delta (LI) = \Delta L \cdot I + \Delta I \cdot L \quad \iff \quad \Delta I = -\frac{\Delta L}{L} I. \]

Magnetic self energy is \( \frac{1}{2} LI^2 \) so

\[ \Delta W_m = \Delta \left( \frac{1}{2} LI^2 \right) = \frac{1}{2} \Delta (LI^2) = \frac{1}{2} \Delta L \cdot I + \frac{1}{2} LI \Delta I = -\frac{1}{2} I^2 \Delta L. \]

Hence the outward force per unit length is

\[ F_m = -\frac{\Delta W_m}{\Delta R} = \frac{1}{2} I^2 \frac{\Delta L}{\Delta R} \frac{1}{2\pi R} = \frac{1}{2} I^2 \frac{dL}{2\pi R} \frac{dR}{dR}. \]

The force is proportional to the derivative of the self-inductance of the loop. \( I \) increases with \( R \)

(2) Pressure force. Derive in the same way using \( \Delta W_p = -p \Delta V \), volume.

How does volume change? \( V = \pi a^2 2\pi R \)

If \( a \) (minor radius) were constant \( V \propto R \).

But a is not constant. The total flux of toroidal mag field is constant. Also \( B_t \propto \frac{1}{R} \)

So

\[ B_t \pi a^2 = \text{const} \propto \frac{a^2}{R} \quad \text{so} \quad a^2 \propto R. \]
Hence in outward motion of plasma \( V \propto R^2 \), \( \Delta V = \frac{2\Delta R}{R} V \).

(per unit length)

The type-tube force on a plasma is thus:

\[
F_r = -\frac{\Delta W_r}{\Delta R} = \frac{1}{2\pi R} \frac{\Delta V}{\Delta R} = \frac{pV}{\pi R^2} = \frac{2\pi a^2}{R} \rho
\]

Total outward self-force is

\[
F_m + F_r = \frac{1}{2} I^2 \frac{dl}{dr} + \frac{2\pi a^2}{R} \rho
\]

The \( \rho \) appearing here is actually the volume average \( \langle \rho \rangle \). So this may be written

\[
F_m + F_r = \frac{\mu_0 I^2}{4\pi R} \left[ \frac{1}{\mu_0} \frac{dl}{dr} + \frac{8\pi a^2}{\mu_0 I_p^2} \langle \rho \rangle \right]
\]

\[
= \frac{\mu_0 I_p^2}{4\pi R} \left[ \frac{1}{\mu_0} \frac{dl}{dr} + \beta_p \right].
\]

The only remaining problem is to calculate the self inductance of the plasma current \( I \).

This has two parts:

The external inductance, from Bfields outside \( r=a \).

The internal inductance, " " inside \( r=a \).

\underline{External inductance} can be calculated in terms of
some nasty elliptic integrals and then approximated
for small \( \frac{a}{R} \). The answer is

\[
L_e = \mu_0 R \left[ \ln \frac{8R}{a} - 2 \right].
\]

(essentially)

One can get the same answer very simply using
a coaxial approximation.

Inductance of a coaxial
cable (per unit length)

\[
L_c = \frac{\psi}{I} = \frac{1}{I} \int_a^b B_0 \, dr
\]

\[
= \frac{1}{I} \int_a^b \frac{\mu_0 I}{2\pi r} \, dr = \frac{\mu_0}{2\pi} \ln \frac{b}{a}
\]

Now think of the toroidal loop as a coax bent round
to join itself. Choose \( b = R \) so that we fill
in the entire middle; and total length is \( 2\pi R \)
So

\[
L_e = 2\pi R \frac{\mu_0}{2\pi} \ln \frac{R}{a} = \mu_0 R \ln \frac{R}{a}.
\]

Note that \( \ln 8 = 2.08 \), so to an excellent
degree of approximation \( \ln \frac{R}{a} = \ln \frac{8R}{a} - 2 \).
Internal Inductance is just related to how much magnetic energy is stored in the magnetic field:

\[
\frac{1}{2} \mathcal{L}_i I_p^2 = \int_0^a \frac{\mathcal{B}^2}{2 \mu_0} 2\pi r \, dr \cdot 2\pi R.
\]

\[
\mathcal{L}_i = 2\pi R \frac{2}{I_p} \frac{1}{2 \mu_0} \int_0^a \mathcal{B}^2 \, 2\pi r \, dr
\]

use \( I_p = \frac{2\pi a \mathcal{B}_0(a)}{\mu_0} \)

\[
= 2\pi R \frac{\mu_0}{4\pi a^2 \mathcal{B}_0(a)} \int_0^a \mathcal{B}^2 \, 2\pi r \, dr
\]

\[
= 2\pi R \frac{\mu_0}{4\pi} \frac{\langle \mathcal{B}_0^2 \rangle}{\mathcal{B}_0^2(a)} = 2\pi R \frac{\mu_0}{4\pi} \mathcal{L}_i
\]

The quantity \( \frac{\langle \mathcal{B}_0^2 \rangle}{\mathcal{B}_0^2(a)} \) has acquired the symbol: \( \mathcal{L}_i \).

This is a dimensionless form of the internal inductance per unit length of the plasma. The extra \( \frac{\mu_0}{4\pi} \) term arises in SI units to convert to practical units. \( \mathcal{L}_i \) just depends on the shape of the current profile. For uniform current density \( \mathcal{L}_i = \frac{1}{2} \).

Now we have the complete plasma inductance:

\[
\mathcal{L} = \mathcal{L}_e + \mathcal{L}_i = \mu_0 R \left[ \ln \frac{8R}{a} - 2 + \frac{\mathcal{L}_i}{2} \right]
\]

from which we calculate \( \frac{d\mathcal{L}}{dR} \) using \( \frac{da}{dR} = \frac{1}{2} \frac{a}{R} \) which follows from \( a^2 \propto R \).
\[
\frac{dl}{dr} = \frac{2l}{3R} + \frac{2l}{3a} \frac{da}{dr} = \mu_0 \left[ \ln \frac{8R}{a} - 2 + i/2 \right] + \mu_0 - \mu_0 \frac{1}{2}
\]
\[
= \mu_0 \left[ \ln \frac{8R}{a} - \frac{3}{2} + \frac{i}{2} \right]
\]

Finally, therefore, we have the expansion self-force per unit length:

\[
F_m + F_p = \frac{\mu_0 I_p^2}{4\pi R} \frac{\left[ \ln \frac{8R}{a} - \frac{3}{2} + \frac{i}{2} + \frac{\beta_p}{2} \right]}{F_m} \frac{I_p}{F_p}
\]

This has to be balanced by a vertical field acting on the plasma current to give an inward \( I_p \times B_v \) force:

\[
I_p \times B_v = F_m + F_p
\]

Hence required vertical field is

\[
B_v = \frac{\mu_0 I_p}{2\pi a} \frac{a}{2R} \left[ \ln \frac{8R}{a} - \frac{3}{2} + \frac{i}{2} + \frac{\beta_p}{2} \right] \sim 2
\]

Vertical field is somewhat smaller \( \left( \frac{a}{2R} \right) \) than \( B_0(a) \) in general.
Stability to Radial Expansion

Equilibrium is

\[ B_v = \frac{\mu_0 I_p}{4\pi R} \left[ \frac{L'}{\mu_0} + \beta_p \right] \]

where ' denotes \( \frac{d}{dR} \). This is the required field.

Stability is determined by

\[ \frac{dB_v}{dR} = B'_v \]

\[ B'_v = \frac{i}{4\pi} \frac{d}{dR} \left[ \frac{I_p L'}{R} + \mu_0 I_p \beta_p \right] \]

\[ = \frac{I_p}{4\pi R} \left\{ (L' + \mu_0 \beta_p) \left( -\frac{i}{R} \right) + (L' + \mu_0 \beta_p) I_p \frac{dI_p}{dR} \right. \]

\[ \left. + L'' + \mu_0 \beta_p' \right\} \]

Now conservation of poloidal flux implies that the equilibrium plasma satisfies:

\[ \frac{d}{dR} (I_p L) = \frac{d}{dR} \left( \int_0^R B_v 2\pi R \, dR \right) \]

which enables \( \frac{dI_p}{dR} \) to be obtained, to substitute in the expression for \( B'_v \). Its value depends on the way in which the external circuits respond to plasma motion.
**Constant Current** (and constant \( i \))

If the tokamak has an iron core, then most of the flux linked by the plasma due to external circuits is in the magnetized core. This tends to have the effect of enforcing constant plasma current, because any plasma current changes give rise to large opposing flux changes.

In this case the required vertical field derivative is

\[
B'_v = \frac{I_p}{4\pi R} \left\{ (L' + \mu_0 B_p) \left( -\frac{1}{R} \right) + L'' + \mu_0 B'_p \right\}
\]

\[
= \frac{I_p (L' + \mu_0 B_p)}{4\pi R} \left\{ -\frac{1}{R} + \frac{L'' + \mu_0 B'_p}{L' + \mu_0 B_p} \right\}
\]

\[
= B_v \left\{ -\frac{1}{R} + \frac{L'' + \mu_0 B'_p}{L' + \mu_0 B_p} \right\}
\]

The marginal field index for stability is then

\[
\eta_c \equiv -\frac{RB'_v}{B_v} = 1 - R \frac{L'' + \mu_0 B'_p}{L' + \mu_0 B_p}
\]

As before, \( L' = \mu_0 \left[ \ln \frac{8R}{a} - \frac{3}{2} + \frac{\zeta}{2} \right] \)

so

\[ L'' = \mu_0 \left[ \frac{1}{R} - \frac{1}{a} \frac{da}{dR} \right] = \frac{\mu_0}{2R} \]
So,
\[ n_c = 1 - \frac{\frac{1}{2} + R\beta'}{\ln \frac{8R}{a} - \frac{3}{2} + \frac{\xi}{2} + \beta_p} \]

How \( \beta_p \) changes with \( R \) depends on the assumptions about energy transport. However, if we assume an adiabatic change, then \( \beta \sim R^{-1/3} \) [Furth & Yoshikawa, Phys. Fluids 13, 2593 (1970)]. Then for constant current

\[ R\beta' = \frac{dR}{dR} \left[ \frac{8\pi a^2 \langle \psi \rangle}{\mu_0 J_p^2} \right] = \beta_p \left\{ \frac{2a'}{a} + \frac{F'}{F} \right\} \]

\[ = \beta_p \left\{ 1 + \frac{-10}{3} \right\} = -\frac{7}{3} \beta_p. \]

Then
\[ n_c = 1 - \frac{\frac{1}{2} - \frac{7}{3} \beta_p}{\ln \frac{8R}{a} - \frac{3}{2} + \frac{\xi}{2} + \beta_p} \]

Since the denominator is typically \( \approx 2 \) to 3, for low \( \beta_p \) the critical index is \( n_c \approx 0.8 \). However, adiabatic pressure changes begin to dominate at \( \beta_p \approx 1 \) and are quite strongly stabilizing. For example, if \( R/a = 3 \), \( \xi = 1 \), \( \beta_p = 1 \), then \( n_c \approx 1.58 \).

[Note that the implicit assumption \( \xi = \text{const.} \) is questionable for the constant current case].
Constant Field

In air-core tokamak the externally produced vertical field depends purely on the coil currents and these are relatively weakly coupled to the plasma. A reasonable approximation is then to take \( \frac{\partial}{\partial t} B_v = 0 \) so that

\[
\frac{d}{dR} \int_R^\infty B_v 2\pi R dR = 2\pi R B_v(R),
\]

and

\[
\frac{dI_p}{dR} = -\frac{I_p}{L} \frac{dL}{dR} + \frac{2\pi R B_v}{L},
\]

\[
= \frac{I_p}{L} \left\{ -L' + \frac{1}{2} \left( L' + M_0 \beta_p \right) \right\}
\]

\[
= \frac{I_p}{L} \left\{ -\frac{L'}{2} + \frac{1}{2} M_0 \beta_p \right\}
\]

Substituting, we find the required

\[
B_v' = \frac{I_p}{4\pi R} \left\{ \left( L' + M_0 \beta_p \right) \left( \frac{1}{R} \right) + \left( L' + M_0 \beta_p \right) \frac{1}{2} \left\{ -\frac{L'}{2} + \frac{M_0 \beta_p}{2} \right\} + L'' + M_0 \beta_p \right\}
\]

\[
= \frac{I_p \left( L' + M_0 \beta_p \right)}{4\pi R} \left\{ -\frac{1}{R} + \frac{-L'}{2L} + \frac{L'' + M_0 \beta_p'}{L' + M_0 \beta_p} \right\}
\]

\[
= B_v \left\{ -\frac{1}{R} + \frac{-L'}{2L} + \frac{L'' + M_0 \beta_p'}{L' + M_0 \beta_p} \right\}
\]

The marginal field index for stability is then

\[ n_c = -\frac{RB_v'}{B_v} \]
\[
nc = 1 - R \left\{ \frac{-L' + \mu_0 \beta_p}{L} + \frac{L'' + \mu_0 \beta_p'}{L' + \mu_0 \beta_p} \right\}
\]

For a circular cross-section plasma, substitute
\[
L' = \mu_0 \left[ \ln \frac{8R}{a} - \frac{3}{2} + \frac{1}{2} \right] \\
L'' = \mu_0 \left[ \frac{1}{R} - \frac{1}{a} \frac{da}{dR} \right] = \frac{\mu_0}{2R} \\
L = \mu_0 R \left[ \ln \frac{8R}{a} - 2 + \frac{1}{2} \right]
\]

Then
\[
nnc = 1 - \left\{ \frac{-\frac{1}{2} \left( \ln \frac{8R}{a} - \frac{3}{2} + \frac{1}{2} \right) + \beta_p/2}{\ln \frac{8R}{a} - 2 + \frac{1}{2}} + \frac{\frac{1}{2} + R \beta'_p}{\ln \frac{8R}{a} - 3 + \frac{1}{2} + \beta_p} \right\}
\]

\[
= 1 + \frac{1}{2} \left[ \ln \frac{8R}{a} - \frac{3}{2} + \frac{1}{2} - \frac{\beta_p}{2} \right] - \left[ \frac{\frac{1}{2} + R \beta'_p}{\ln \frac{8R}{a} - 3 + \frac{1}{2} + \beta_p} \right]
\]

The result for this case is often taken to be \( n_{nc} = 1.5 \). This is exact only in the (extreme) limit \( \ln 8R/a \to \infty \). When we are not in that limit, we need \( \beta'_p \), which for the adiabatic assumption is \( R \beta'_p = -\frac{7}{3} \beta_p \) (see constant current case).

Then, for example, if \( R/a = 3 \), \( l_i = 1 \),
\( \beta_p = 0 \) \( \Rightarrow \) \( n_{nc} = 1.42 \)
\( \beta_p = 1 \) \( \Rightarrow \) \( n_{nc} = 1.63 \) \( [\text{Note, however, that the third term contributes +0.58 to } n_{nc} \text{ in this case}] \).
Graphical View of Equilibrium Field

$B_{\text{plasma}}$

$\uparrow$ $\downarrow$

$B_v$

$\rightarrow$

Weaker Stronger

Weaker Stronger

$B_p$: Stronger Weaker

Plasma Field Alone.

Vertical field makes $B_p$ stronger on outside to balance $\nabla p$ forces.

If $B_v$ is made strong enough, a separatrix appears at the point where $B_v = -B_{\text{plasma}}$.

Strong $B_v$

Strongest $B_v$ (Equilibrium Limit.)

As $B_v$ gets stronger x-point moves toward plasma till it reaches plasma edge and begins to shrink the plasma region.
Why would $B_v$ get bigger? Answer: because $\beta_p$ (plasma pressure) was increased.

Consequence: There is a maximum allowable $\beta_p$ before the separatrix invades the plasma. This is the equilibrium limit.

Simple minded estimate:

For large $\beta_p$, $B_v \approx \frac{\mu_0 I_p}{2\pi a} \frac{a}{2R} \beta_p$

x-point is where $|B_v| = |B_{\text{plasma}}|$

If we take $B_{\text{plasma}} = \frac{\mu_0 I_p}{2\pi a}$ we would get x-point at plasma edge when $\beta_p = \frac{2R}{a} = \frac{2}{\epsilon}$. [Not quite correct]

Actually, $B_{\text{plasma}}$ is smaller than $\frac{\mu_0 I_p}{2\pi a}$ on the inboard side.

And in fact on inboard side $|B_{\text{plasma}}| \approx |\frac{\mu_0 I_p}{2\pi a}| - |B_v|$

[not proved here but can be]

In that case, x-point is at plasma edge when $2|B_v| = \frac{\mu_0 I_p}{2\pi a}$

Internal Magnetic Fields:

Shafranov Shift.

Current shifted outward
Thus equilibrium beta limit is \((\beta_0 \leq \frac{1}{c})\) or:

\[ e \beta_0 \approx 1. \]  \[ \text{[Circular cross-section]} \]

This is only approximate for lots of reasons.

In particular, we don't have to make \(B_v\) uniform in space. If we made it stronger on the outboard side and weaker on the inboard, that helps to prevent the separatrix from entering.
Tokamak Ordering

Call the inverse aspect ratio $\frac{a}{R} = \varepsilon$.

It is implicit in what we have done that $\varepsilon$ is small. [e.g. $B_0 = \mu_0 I / 2\pi r$ is approximate, valid for $\varepsilon \ll 1$].

The safety factor is the more precise way of saying that the toroidal field is strong:

$$q = \frac{r B_r}{RB_\theta} \geq 1$$

This is for stability (see later).

Then $\frac{B_\theta}{B_\phi} \approx \frac{r}{R} \frac{1}{q} \sim \varepsilon/q \leq \varepsilon$.

Consequence is that total (or toroidal) beta is much smaller than poloidal:

$$\beta_t = \frac{2\mu_0 \langle \rho \rangle}{B_t^2} = \frac{B_\theta^2}{B_t^2} \beta_\phi \sim \frac{\varepsilon^2}{q^2} \beta_\phi.$$  

We distinguish two regimes of Tokamak operation: Low $\beta$ & High $\beta$  
[Sometimes: ‘Ohmic’ & High $\beta$]
These are distinguished by the value of $\beta_p$. Tokamaks with only ohmic heating tend to have $\beta_p \leq 1$ [e.g. Alcator C $\beta_p \sim 0.3$]; the low $\beta$ ordering is $\beta_p \sim 1$. Strongly auxiliary heated plasmas can approach the beta limit $\beta_p \sim \frac{1}{\epsilon}$: high beta ordering.

<table>
<thead>
<tr>
<th>Low Beta</th>
<th>High Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse Aspect Ratio</td>
<td>$\epsilon = \frac{a}{R}$</td>
</tr>
<tr>
<td>Safety factor, $q$</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>Field Ratio, $\frac{B_0}{B_\phi}$</td>
<td>$\sim \frac{\epsilon}{q}$</td>
</tr>
<tr>
<td>Poloidal Beta, $\beta_p$</td>
<td>$\leq 1$</td>
</tr>
<tr>
<td>Toroidal Beta, $\beta_r$</td>
<td>$\sim \frac{\epsilon^2 \beta_p}{q^2}$</td>
</tr>
</tbody>
</table>

[Diagrams showing paramagnetic and diamagnetic behavior]