Introduction

Given all we know about the constraints on operations of a tokamak, and some sort of confinement scaling law, we can predict the performance of the tokamak under assumed conditions.

This prediction is a “Zero-dimensional” calculation, in the sense that we don’t calculate the profiles self consistently. Instead we just assume that they are given. Then the shape of (e.g.) the $T_e$ profile is fixed by our assumptions but its magnitude is to be solved for self-consistently with power balance.

These estimates do not solve for particle balance. They just assume (not necessarily realistically) that the density can be controlled somehow. A range of densities is explored to see whether there is some optimum.

Machine Parameters

The things that determine the capabilities of a tokamak are basically its size, shape, and magnetic field.

- Major Radius (m) $R$
- Minor Radius (m) $a$
- Elongation $\kappa$
- Toroidal Magnetic Field (T) $B_T$
- Edge Safety Factor $q_a$

Of course, there are many other aspects of a tokamak that appear in (e.g.) scaling laws. These include very importantly $I_p$. But $I_p$ can be derived from knowing the above parameters and the way $q_a$, $B_T$, $R$, $a$, and $I_p$ are related:

$$q_a = \frac{2\pi a^2 B_T}{R\mu_0 I_p} \left( \frac{1 + \kappa^2}{2} \right)$$

[I’m using $q_\psi$ formula for $q_a$.]

Power Balance Equations

Zero-D electron power balance:

$$P_e - P_{ei} = W_e/\tau_{ee}$$

(2)
and ion power balance:

\[ P_i + P_{ei} = W_i / \tau_{Ei} \]  \hspace{1cm} (3)

must be solved.

The electron to ion equilibration power \( P_{ei} \) is calculated from Coulomb collisions

\[ P_{ei} = C_2 n_e (T_e - T_i) / T_e^{3/2} \]  \hspace{1cm} (4)

with \( C_2 \) a coefficient (dependent on \( \ln \Lambda \) but we take \( \ln \Lambda \) to be simply a constant typical value).

Using profiles of the form \( (1 - r^2/a^2)^{\gamma} \) we e.g. express average energy-density in terms of central energy density \( W \) as \( 3n_0 T_0 / (1 + \gamma_n + \gamma_T) \); and similarly for other quantities, so everything is expressed in terms of the central values of the power- or energy-density. There are several places where heuristic approximations have to be adopted with respect to profiles. \( P_{ei} \) is one such case in point.

**Confinement/Transport scalings**

Then we can choose to explore various different assumptions about the confinement, to see what performance would be for a particular tokamak under that confinement law. Various options include:

**NeoAlcator Scaling**

\[ \tau_{NA} = 2 \times 10^{-21} n a R^2 \kappa^{0.5} \]  \hspace{1cm} (5)

**Goldston scaling**

\[ \tau_G = 0.037 I R^{1.75} a^{-0.37} \kappa^{0.5} P_t^{-0.5} \]  \hspace{1cm} (6)

**ITER89p scaling**

\[ \tau_{89p} = 0.048 I_p^{0.85} R^{1.2} a^{0.3} B_T^{0.2} n^{-0.1} \kappa^{0.5} \mu^{0.2} P_t^{-0.5} \]  \hspace{1cm} (7)

**ITER97 scaling**

\[ \tau_{97} = 0.073 I_p^{0.9} R^{1.84} a^{0.2} B_T^{0.2} n^{0.4} \kappa^{0.92} \mu^{0.2} P_t^{0.66} \]  \hspace{1cm} (8)

**ITER98y2 scaling**

\[ \tau_{98y} = 0.145 I_p^{0.93} R^{1.39} a^{0.58} B_T^{0.15} n^{-0.41} \kappa^{0.78} \mu^{0.19} P_t^{0.69} \]  \hspace{1cm} (9)

These losses are considered to be multiplied by a corresponding weighting factor \( f \) and then apportioned to electrons and ions by a chosen fraction \( L_e \) so that the electron loss is governed by

\[ 2 \tau_{Ee} L_e = \left( (f_{NA}/\tau_{NA})^2 + (f_G/\tau_G)^2 + (f_{89p}/\tau_{89p})^2 \right)^{-1/2} \]  \hspace{1cm} (10)

and radiation losses are also added.

For the ions, Neoclassical confinement time losses

\[ \tau_{NC} = 0.07 I^{2} R^{0.5} a^{-0.5} n^{1} T_i^{0.5} Z^{-1} \mu^{-0.5} / K_2 \]  \hspace{1cm} (11)

(where \( K_2 \) is the Chang-Hinton coefficient \( \sim 0.5 \)) are added, so that

\[ 1/\tau_{Ei} = 1/\tau_{NC} + (1 - L_e)/(L_e \tau_{Ee}) \]  \hspace{1cm} (12)
The units in all these formulas are MA, meters, Tesla, keV, $10^{30} \text{m}^{-3}$, MW, and confinement times in seconds.

We prescribe a certain total amount of auxiliary heating to electrons and to ions. Non-Bremsstrahlung radiated power and ohmic heating consistent with the temperature are applied to electrons. Some combination of the transport loss factors $f_{...}$ is adopted and then the power balance equations (2,3) are solved by an iteration scheme (based on Newton’s method). The result is to find $T_e$ and $T_i$ and the other parameters of the plasma when in power balance equilibrium, such as $\tau_E$ (which might depend upon $T$). Fig. 1 shows an example Lawson diagram ($n_0\tau_E$ versus $T_i$) with performance along a diagonal line for various different values of the density. The corresponding variation of $\tau_E$ and $T$ with density is shown in Fig. 2.

**Popcon Performance Contours**

As an alternative to solving for temperature given density and heating, one can turn the problem around and solve for required heating, given temperature and density. Performing this calculation over a two-dimensional array of temperatures and densities leads to a plot that is traditionally called “popcon” showing performance operating contours. [Originally

![Figure 1: Performance calculation for the EDA version of ITER, with 60MW of total heating and a range of densities.](image-url)
these were calculated on a one-fluid approximation (combined electrons and ions). The present calculation is a more sophisticated one based on allowing $T_e \neq T_i$. These plots are very helpful for understanding the predicted performance of hypothetical tokamaks that are intended to have substantial fusion-alpha heating power. In other words, burning plasmas or ignition devices. The quantity then contoured is the required amount of non-fusion auxiliary power required to operate the tokamak at a particular point in density-temperature space. Figure 3 shows an example, where the parameters of the EDA ITER device are taken.

On the left-hand side of the plot, where the temperature is low, the power contours are negative. One would have to apply auxiliary cooling to operate the tokamak there, because ohmic heating alone is sufficient to raise the temperature at any given density to the position of the ohmic heating zero-power contour, typically a few keV.

Immediately to the right of that power contour the auxiliary power requirement is positive. The plasma requires auxiliary heating to operate there. As one moves farther to the right at constant intermediate density (e.g. $0.8 \times 10^{20} \text{m}^{-3}$), the power requirement rises, reaches a peak (ridge) and then falls again until another zero power contour is reached. The reason for the fall in power is that alpha-particle fusion heating takes over the role from the auxiliary heating and eventually becomes sufficient to sustain the plasma by itself. This second zero power contour is the ignition line.

If the auxiliary power were turned off just to the right of the ignition contour, then the plasma would continue to heat up, because it would have more alpha-particle heating than required for steady state. It would continue to move right, to higher temperature, till it
finally encountered the zero power contour again. It would then reach a steady equilibrium, but at very high temperature.

Similarly, an unheated plasma that started just to the left of the ignition contour would cool (because it had insufficient heating). It would continue to cool until it met the ohmic-heating zero-power contour. The ignition contour is thus unstable to temperature perturbations in either direction (when the power requirement gradient component in the $T$-direction is negative).

Various other criteria are also contoured on this plot. The place where $Q = 10$ is the target circulating power quality factor for (the current) ITER. The 100MW loss contour is where a device with 500MW total fusion power would operate. The Greenwald density limit is a horizontal line. $P/nBS = 0.02$ is one scaling criterion for the minimum power to obtain H-mode. Also beta=$3I/aB$ is the Troyon beta limit.

Other parameters can be extracted from the calculations and contoured over the same domain. Fig. 4 shows three quantities. The electron temperature contours are mostly vertical lines because the power balance requires that there not be very much difference between $T_e$ and $T_i$ at reasonable densities when confinement is sufficient for fusion burn. The radiation $P_{rad}$ is here just Bremsstrahlung, though with $Z_{eff} = 1.5$. It is not very significant compared with the heating and alpha powers. And $P_{alpha}$ shows how the alpha heating varies over the domain.
Figure 4: Other power parameters over the popcon domain.

**Trying out the program**

Go to an athena (Linux) workstation or ssh to athena:

```
ssh athena.dialup.mit.edu
```

and log in.

Get the program to your directory, unpack it and build it:

```
$ cp /afs/athena.mit.edu/user/i/h/ihutch/genc.zip .
$ unzip genc.zip
$ cd gencpackage
$ make
```

Run it with (e.g.)

```
$ ./genc iterh2.dat
```

Hack around inside your versions of the input files to change the parameters and run again. Graphics will display provided you have the ability to display X. Figures like `plot0001.ps` will be left behind in the directory you run the program from.

If you want to improve the program. Edit `genc.f` and type make again. Send me patches if you make substantial upgrades. The program is a bit of a hack at the moment.