Stellarator Confinement

In the same spirit as tokamak scaling. Translate using

$$\frac{1}{Q_s} = t = \frac{M_0 R I_p}{2\pi a^2 B_c}$$

[circular tokamak expression]

In these terms, ITER98(y2) has

$$T_e \propto R^{2.62} 0.46 \quad 0.93 \quad 1.08 \quad 0.41$$

Quite similar to:

$$T_e^{18895} = 0.079 a_{eff}^{2.21} R^{0.65} t_{2/3}^{0.4} B_c^{0.83} n_{19}^{0.51} P_{MW}^{-0.59} s$$

$$T_e^{18804} = f 0.134 a_{eff}^{2.28} R^{0.64} t_{2/3}^{0.41} B_c^{0.84} n_{19}^{0.54} P_{MW}^{-0.61} s$$

[a_{eff} is an effective minor radius. \( n_{19} = N_e / 10^{19}m^{-3} \).

f is a 'configuration' factor \( \sim 1 \) for best, but degrading to 0.5]

The $T_e^{18895}$ was found to predict LHD confinement rather well, giving substantial confidence.

Stellarators do not seem to have a clear density limit.

LHD has achieved (separately)

$$N_e(0) = 1.2 \times 10^{21} m^{-3} \quad 1.5 \text{ Bar pressure} \quad T_i = 7.5 \text{ keV}$$

$$\langle \beta \rangle = 5\% \quad 1 \text{ keV plasma for 1 hour}$$
Stellarator Optimization

Since it is 3-dimensional a stellarator has many more options for optimization than a tokamak (2-D).

Probably the biggest concern (at least currently) is the confinement of (energetic) single-particles when trapped. Optimizing this aspect has been a major focus.

Trapped Particles Occur when there is a local minimum in $|B|$

Example 1. Axisymmetric Tokamak

Particles are trapped on outbound side. Banana orbits are wide but still localized near $\psi$-surface.

Example 2. Tokamak with Ripple.

If a tokamak has major toroidal ripple a new class of ripple-trapped orbits arises.

Ripple-trapped particles are not localized to a $\psi$-surface. They can drift right out of the plasma.

Well-designed tokamaks avoid significant ripple.
3-D Stellarators generally have substantial ripple and ripple-trapped lost or transported ions. (n,m-periods)
A particular concern for fusion is energetic α-particles.
A stellarator configuration that avoids this problem so that all collisionless trajectories are confined is called omnigenous. This requires the average grad-B & curvature drift V_d in the direction of \( \nabla \psi \) to be zero.

One way to achieve omnigenicity is to have a field configuration in which the field-magnitude is a function only of flux-surface, \( \psi \), and a unique combination of the in-surface angular coordinates \( \chi, \zeta \) such that \( B = B(\psi, M_\chi - N_\zeta) \). Such configurations are said to be quasi-symmetric.

Quasi-symmetric configurations are a subset of omnigenous configurations.

Flux-surface angular coordinates

Suppose there are simply-nested toroidal flux surfaces. We can specify position on any surface via 2 coordinates.
E.g. tokamak \( \theta \) poloidal \( \phi \) toroidal
Since surface is a toroid, \( \theta, \phi \) are periodic. Choose period to be \( 2\pi \).
Within the constraints of periodicity, there is an infinity of choices of equivalent angular coordinates. Call the alternative to \((\theta, \phi)\) \((\chi, s)\).

There is a special choice of \((\chi, s)\) called "Boozer Coordinates" in which the guiding-center drift (in the \(\Psi\) direction) depends only on constants of the motion \((E & \mu)\), flux \(\Psi\), and magnitude of \(B\).

In this coordinate system, if there is an ignorable coordinate, whose variation causes no variation in \(B\), then a conserved quantity is associated with it.

[Analogy: In a system with translational symmetry (indep of \(z\)) the momentum \(mv_z\) is conserved.]

The conserved quantity forces the drift orbit to remain close to the flux-surface.

Quasi-symmetry is the presence of an ignorable coordinate (for \(181\)).

**Quasi-poloidal symmetry** \[ B = B(\psi, s) \] \(\chi\)-ignorable. [W7X]

**Quasi-toroidal symmetry** \[ B = B(\psi, \chi) \] \(s\)-ignorable. [NCSX]

**Quasi-helical symmetry** \[ B = B(\psi, N\chi - Ns) \] [HSX]
Requirement for Omnigenicity.

All B-contours must topologically link the flux surface. Otherwise, there's a local max or min, and hence, for some field-line $B \cdot \nabla B$ is non-zero at the B-minimum (or B-max). Deeply trapped (marginally) will then drift far from surface there.

The minima & maxima of B must all be at the same height. (Valley, ridge).

Magnetic field lines must never be tangent to B-contours.

A quasi-symmetric configuration has B-contours that are parallel straight-lines.

Trapped particles drift along the valleys.
W7-X Quasi Poloidal Symmetry

Field magnitude is (ideally) invariant around a poloidal transit.

Like a series of toroidally-linked simple mirrors.

But... simple mirrors are unstable because of bad curvature.

Also, toroidal force balance w/o parallel current is subtle.

"Minimizing j_i" makes the equilibrium most insensitive
to changes in pressure $\frac{dP}{dy}$.

Inexact symmetry.

Emphasizing B min region.

Superconducting coils

Designed 1990s

Many difficulties with construction. Completion expected 2014.
NCSX

Quasi Axisymmetry
Torusoidal Symmetry

Has substantial net toroidal current. So can be considered a tokamak hybrid.
100% Bootstrap.

More compact
lower aspect ratio
Rotational transform from Ip can approach helical transform.

Configuration evolves much in going from p=0 Ip=0 to its high pressure state.

Massive escalation in construction costs.